$$
\begin{aligned}
w_1 &= (9\cdot x\cdot x\cdot x\cdot x - y^4) + 2\cdot y\cdot y,\\
w_2 &= (3\cdot x\cdot x - y\cdot y)\cdot(3\cdot x\cdot x + y\cdot y) + 2\cdot y\cdot y,\\
w_3 &= 9x^4 + (2\cdot y\cdot y - y\cdot y\cdot y\cdot y),\\
w_4 &= (9x^4 + 2\cdot y\cdot y) - y^4.
\end{aligned}
$$

$$\begin{aligned}
w_1 &= -9.990137 \cdot 10^{12}, \\
w_2 &= 9.863382 \cdot 10^{9}, \\
w_3 &= 0.000000, \\
w_4 &= -1.000000 \cdot 10^{13}.
\end{aligned}$$

$$
\begin{aligned}
w_1 &= -9.366178 \cdot 10^8, \\
w_2 &= 1.000000 \cdot 10^0, \\
w_3 &= 1.800000 \cdot 10^9, \\
w_4 &= 8.000000 \cdot 10^8.
\end{aligned}
$$

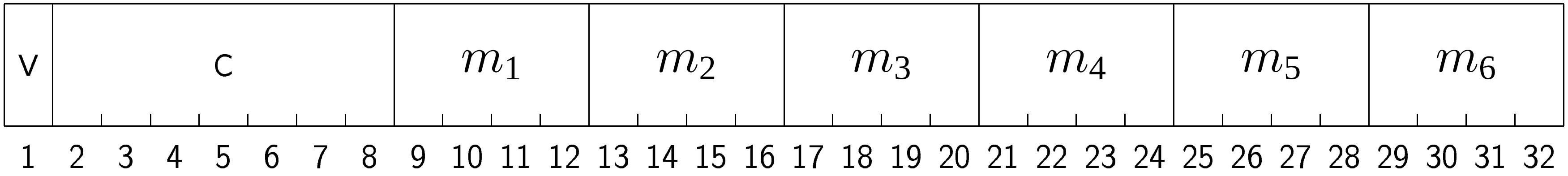
$$\begin{array}{rcll} & & 7-stellig & 12-stellig \\ w_1 & = & 2.161918 \cdot 10^{20} & 2.161918 \cdot 10^{20}, \\ w_2 & = & 2.161917 \cdot 10^{20} & 2.161918 \cdot 10^{20}, \\ w_3 & = & 2.161918 \cdot 10^{20} & 2.161918 \cdot 10^{20}, \\ w_4 & = & 2.161918 \cdot 10^{20} & 2.161918 \cdot 10^{20}. \end{array}$$

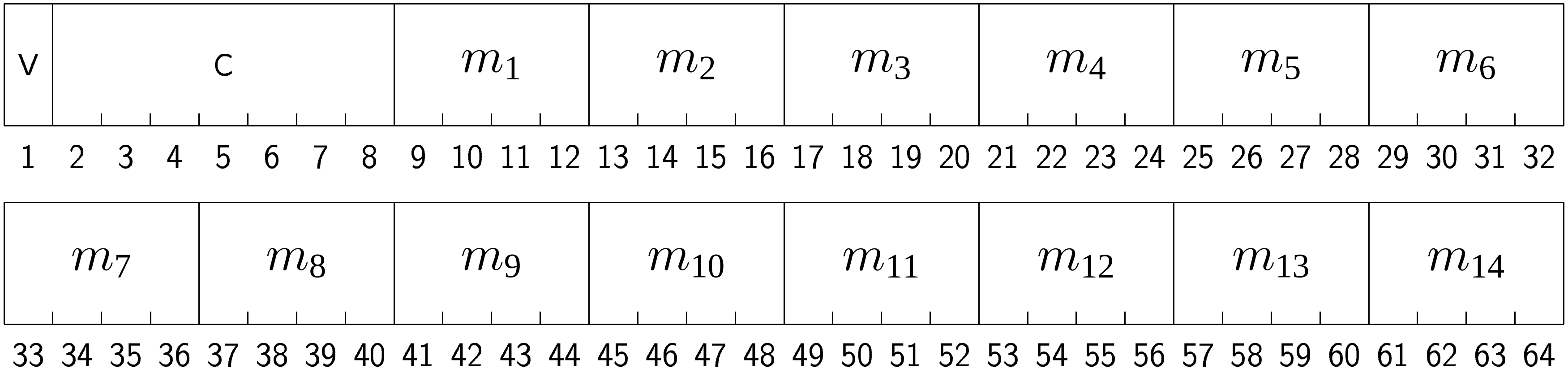
$$\begin{array}{ccccccccc} x_1 & + & \frac{1}{2}x_2 & + & \frac{1}{3}x_3 & + & \frac{1}{4}x_4 & = & 1 \\ \frac{1}{2}x_1 & + & \frac{1}{3}x_2 & + & \frac{1}{4}x_3 & + & \frac{1}{5}x_4 & = & 1 \\ \frac{1}{3}x_1 & + & \frac{1}{4}x_2 & + & \frac{1}{5}x_3 & + & \frac{1}{6}x_4 & = & 1 \\ \frac{1}{4}x_1 & + & \frac{1}{5}x_2 & + & \frac{1}{6}x_3 & + & \frac{1}{7}x_4 & = & 1 \end{array}$$

	$4-stellig$	$5-stellig$	$6-stellig$	$8-stellig$
$x_1 =$	-5.8999	-4.1814	-4.0262	-4.0003
$x_2 =$	80.5437	61.9951	60.2963	60.0033
$x_3 =$	-228.5033	-184.7562	-180.7181	-180.0080
$x_4 =$	171.1528	143.0748	140.4694	140.0052

n	$I_n\,(7-stellig)$	$I_n\,(16-stellig)$
0	1.71828	1.71828
2	0.43656	0.43656
4	0.23877	0.23876
6	0.16304	0.16292
8	0.13024	0.12332
10	0.72160	0.09911
12	82.2512	0.08281
14	14954.72	0.07110
16		0.06506
18		0.90685
20		323.60412
20		149482.10144

n	$I_n\,(7-stellig)$	$I_n\,(16-stellig)$
0	1.71828	1.71828
2	0.43656	0.43656
4	0.23876	0.23876
6	0.16290	0.16290
8	0.12222	0.12222
10	0.0	0.0


V
C
m_1
m_2
m_3
m_4
m_5
m_6
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32


V
C
m_1
m_2
m_3
m_4
m_5
m_6
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32
m_7
m_8
m_9
m_{10}
m_{11}
m_{12}
m_{13}
m_{14}
33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64

Absolute und relative Konditionszahlen elementarer Funktionen

$f(x)$	$\lvert C(f)\rvert = \lvert f'(x)\rvert$	$\lvert c(f)\rvert = \left\lvert \frac{x\cdot f'(x)}{f(x)} \right\rvert$
$x^\alpha(\alpha \in \mathbb{R}_+)$	$\alpha\lvert x\rvert^{\alpha-1}$	α
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	$\frac{1}{2}$
x^{-1}	$\frac{1}{x^2}$	1
$\ln x$	$\frac{1}{\lvert x\rvert}$	$\frac{1}{\lvert \ln x\rvert}$
e^x	e^x	$\lvert x\rvert$
$\sin x$	$\lvert \cos x\rvert$	$\lvert x \cot x\rvert$
$\cos x$	$\lvert \sin x\rvert$	$\lvert x \tan x\rvert$
$\tan x$	$\frac{1}{\cos^2 x}$	$\left\lvert \frac{2x}{\sin 2x} \right\rvert$

Koeffizienten der baryzentrische Darstellung

Input: *Stützstellen* $x_0, \ldots, x_n$.
Output: *Koeffizienten der baryzentrischen Darstellung*

```
for $i = 0$ to $n$ do
    $t = 1$; $s = x_i$
    for $k = 0$ to $i - 1$ do
        $t := t \cdot (s - x_k)$
    endfor
    for $k = i + 1$ to $n$ do
        $t := t \cdot (s - x_k)$
    endfor
    $a_i = 1/t$
endfor
```

Interpolationswert bei baryzentrischer Darstellung

Input: *Stelle x, Stützpunkte $(x_0, f_0), \ldots, (x_n, f_n)$ und Koeffizienten $a_0, \ldots, a_n$ der baryzentrischen Darstellung*
Output: *Interpolationswert an der Stelle x.*

```
  $b_0 = \frac{a_0}{x - x_0}$; $s_1 = b_0 \cdot f_0$; $s_2 = b_0$
  for $i = 1$ to $n$ do
    $b_i = \frac{a_i}{x - x_i}$; $s_1 = s_1 + b_i \cdot f_i$; $s_2 = s_2 + b_i$
  endfor
  $P = \frac{s_1}{s_2}$
```

Neville-Algorithmus

```
Input: Stelle $\bar{x}$ und Stützpunkte $(x_0, f_0), \ldots, (x_n, f_n)$
Output: Interpolationswert an der Stelle $\bar{x}$
    for $i = 1$ to $n$ do
        $P_{i0} = f_i$
        for $k = 1$ to $i$ do
            $P_{ik} = \frac{(\bar{x} - x_{i-k})P_{i,k-1} - (\bar{x} - x_i)P_{i-1,k-1}}{x_i - x_{i-k}}$
                $= P_{i,k-1} + \frac{\bar{x} - x_i}{\bar{x} - x_{i-k}}\left[P_{i,k-1} - P_{i-1,k-1}\right]$
                $= P_{i,k-1} + \frac{P_{i,k-1} - P_{i-1,k-1}}{\frac{\bar{x} - x_{i-k}}{\bar{x} - x_i} - 1}$
        endfor
    endfor
```

	$k = 0$	1	2	3	4	5	6	7
$\bar{x} - x_0$	P_{00}							
		P_{11}						
$\bar{x} - x_1$	P_{10}		P_{22}					
		P_{21}		P_{33}				
$\bar{x} - x_2$	P_{20}		P_{32}		P_{44}			
		P_{31}		P_{43}		P_{55}		
$\bar{x} - x_3$	P_{30}		P_{42}		P_{54}		P_{66}	
		P_{41}		P_{53}		P_{65}		P_{77}
$\bar{x} - x_4$	P_{40}		P_{52}		P_{64}		P_{76}	
		P_{51}		P_{63}		P_{75}		
$\bar{x} - x_5$	P_{50}		P_{62}		P_{74}			
		P_{61}		P_{73}				
$\bar{x} - x_6$	P_{60}		P_{72}					
		P_{61}						
$\bar{x} - x_7$	P_{70}							

Auswertung NEWTONsches Interpolationspolynoms

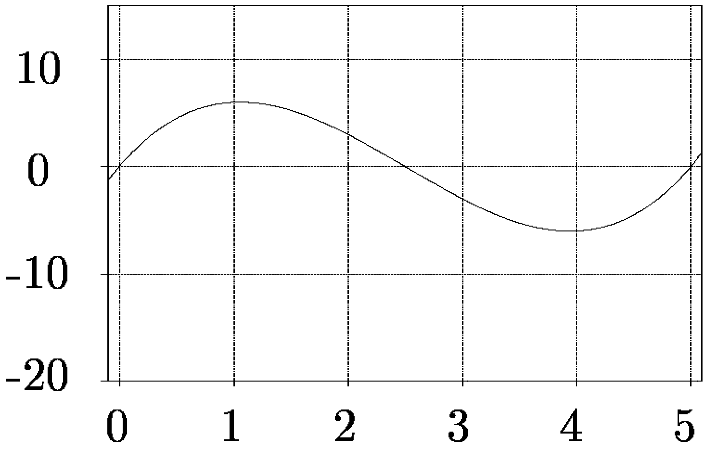
Input: *Stelle* $\overline{x}$,*Stützstellen* $x_0,\ldots,x_n$ *und die Koeffizienten* $a_0,\ldots,a_n$ *des* NEWTON*schen Interpolationspolynoms*

Output: *Interpolationswert an der Stelle* $\overline{x}$.

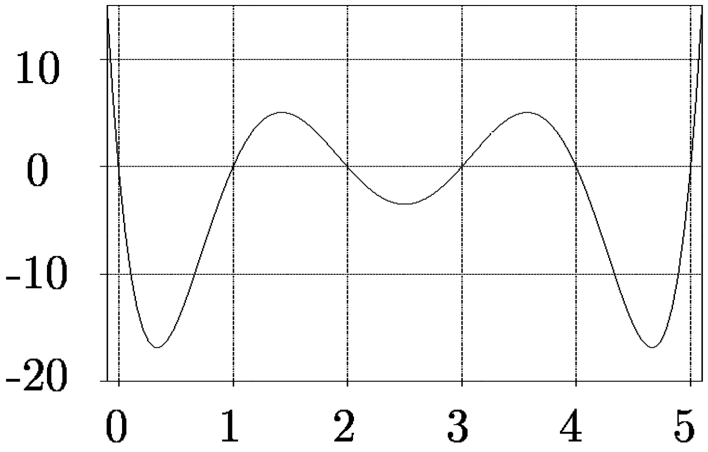
```
$P = a_n$
for $i = n-1$ to $0$ step $-1$ do
  $P = a_i + (\overline{x} - x_i) \cdot P$
endfor
```

Aufwand: n Multiplikationen und $2n$ Additionen.

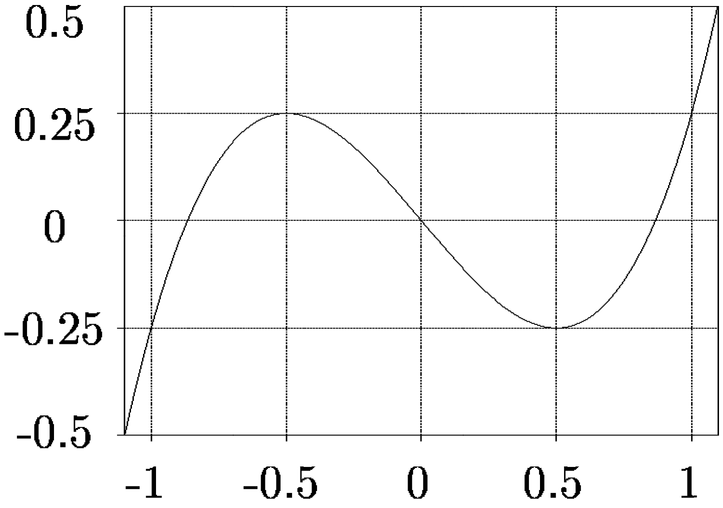
	$k=0$	1	2	3	4
x_0	$f_0 = f[x_0]$				
		$f[x_0, x_1]$			
x_1	$f_1 = f[x_1]$		$f[x_0, x_1, x_2]$		
		$f[x_1, x_2]$		$f[x_0, x_1, x_2, x_3]$	
x_2	$f_2 = f[x_2]$		$f[x_1, x_2, x_3]$		$f[x_0, x_1, x_2, x_3, x_4]$
		$f[x_2, x_3]$		$f[x_1, x_2, x_3, x_4]$	
x_3	$f_3 = f[x_3]$		$f[x_2, x_3, x_4]$		
		$f[x_3, x_4]$			
x_4	$f_4 = f[x_4]$				



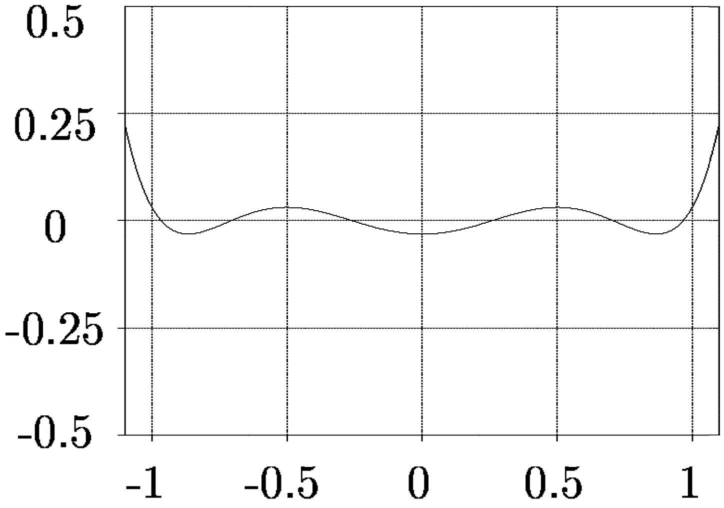
$\omega(x) = x(x-2.5)(x-5)$



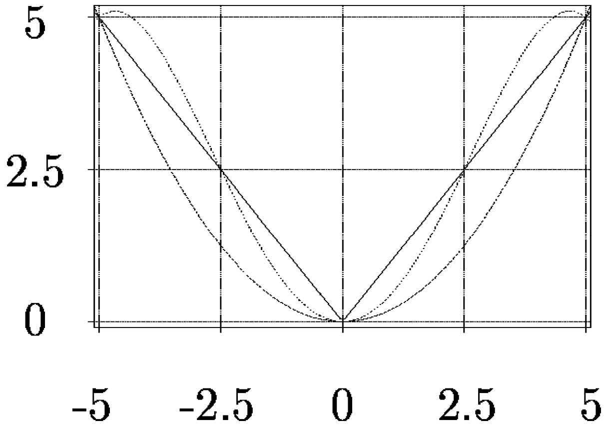
$\omega(x) = x(x-1)(x-2)(x-3)(x-4)(x-5)$

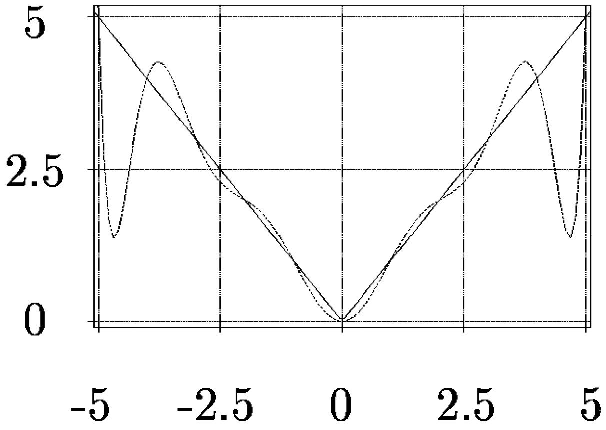


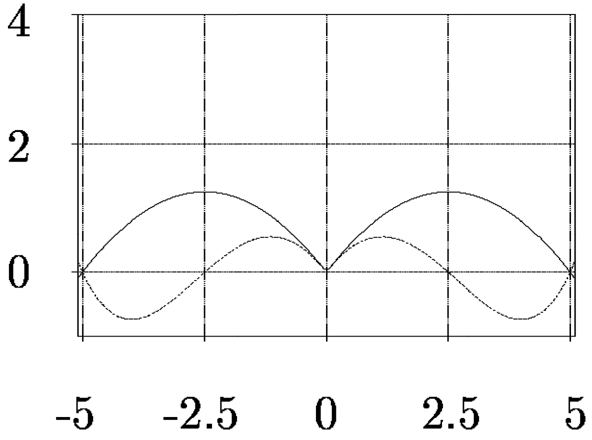
$\omega(x) = T_2(x)$

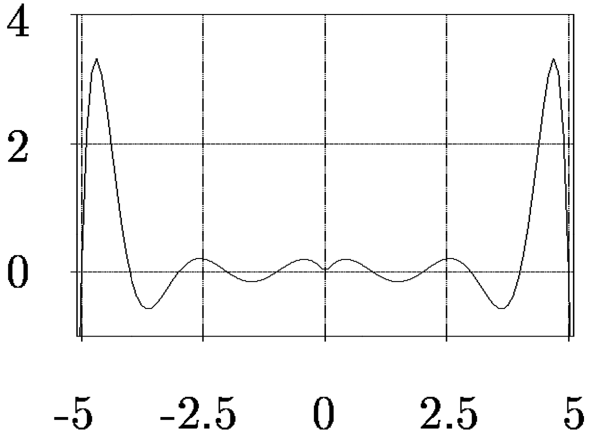


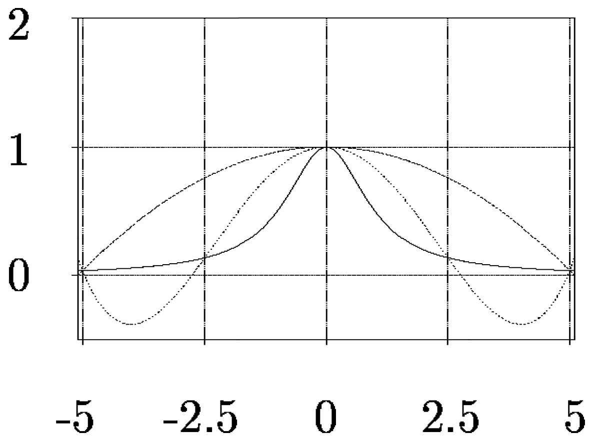
$\omega(x) = T_5(x)$

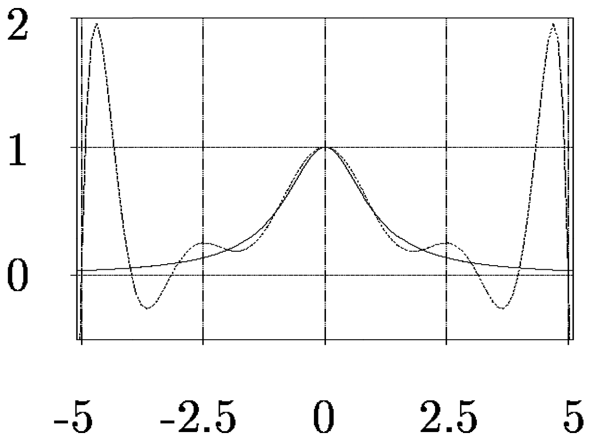

5
2.5
0
-5
-2.5
0
2.5
5

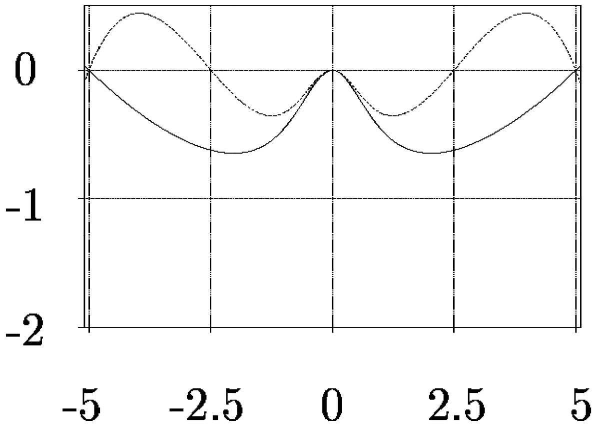

5
2.5
0
-5
-2.5
0
2.5
5

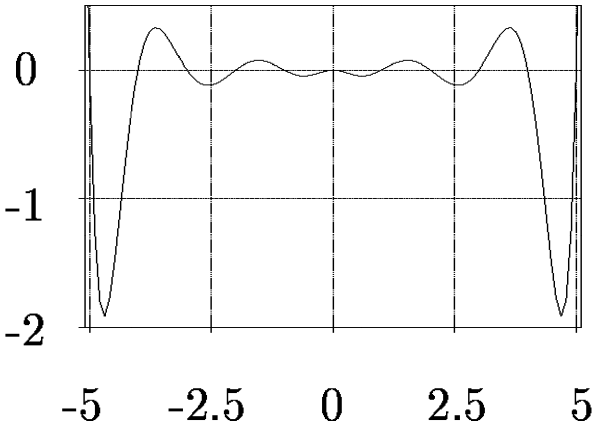

4
2
0
-5
-2.5
0
2.5
5


4
2
0
-5
-2.5
0
2.5
5


2
1
0
-5
-2.5
0
2.5
5


2
1
0
-5
-2.5
0
2.5
5


0
-1
-2
-5
-2.5
0
2.5
5


0
-1
-2
-5
-2.5
0
2.5
5

STOER-Algorithmus

Input: *Stelle $\overline{x}$ und Stützpunkte $(x_0, f_0), \ldots, (x_n, f_n)$.*
Outut: *Wert einer interpolierenden rationalen Funktion an der Stelle $\overline{x}$.*

```
for $i = 1$ to $n$ do
    $T_{i0} = f_i$
endfor
for $k = 1$ to $i$ do
    $T_{ik} = T_{i,k-1} + \frac{T_{i,k-1} - T_{i-1,k-1}}{\frac{\bar{x} - x_{i-k}}{\bar{x} - x_i}\left[1 - \frac{T_{i,k-1} - T_{i-1,k-1}}{T_{i,k-1} - T_{i-1,k-2}}\right] - 1}$
endfor
```

	$j=0$	1	2	3	4	5
	$f_0=T_{00}$					
$0=T_{0,-1}$		T_{11}				
	$f_1=T_{10}$		T_{22}			
$0=T_{1,-1}$		T_{21}		T_{33}		
	$f_2=T_{20}$		T_{32}		T_{44}	
$0=T_{2,-1}$		T_{31}		T_{43}		T_{55}
	$f_3=T_{30}$		T_{42}		T_{54}	
$0=T_{3,-1}$		T_{41}		T_{53}		
	$f_4=T_{40}$		T_{52}			
$0=T_{4,-1}$		T_{51}				
	$f_5=T_{50}$					

Auswertung eines Kettenbruchs

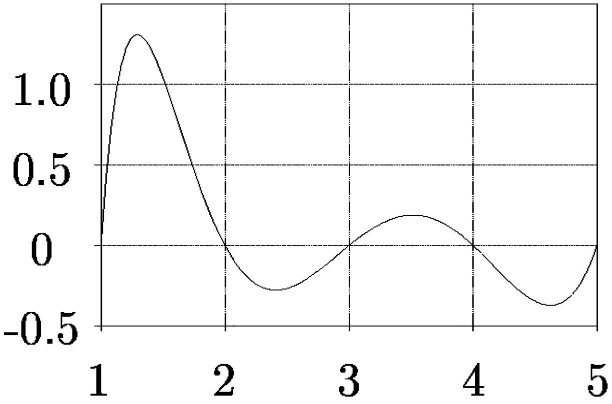
Input: *Stelle* $\bar{x}$ *und Stützpunkte* $(x_0, f_0), \ldots, (x_n, f_n)$.
Outut: *Wert des Kettenbruchs*

$$\Phi(x) = a_0 + \underline{x - x_0}\big/\overline{a_1} + \underline{x - x_1}\big/\overline{a_2} + \cdots + \underline{x - x_{n-1}}\big/\overline{a_n}$$

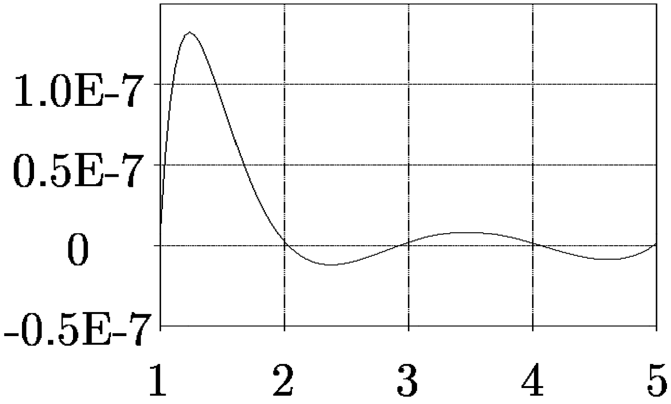
an der Stelle $\bar{x}$.

```
$\Phi = a_n$
for $i = n-1$ to $0$ step $-1$ do
    $\Phi = a_i + (\bar{x} - x_i)/\Phi$
endfor
```

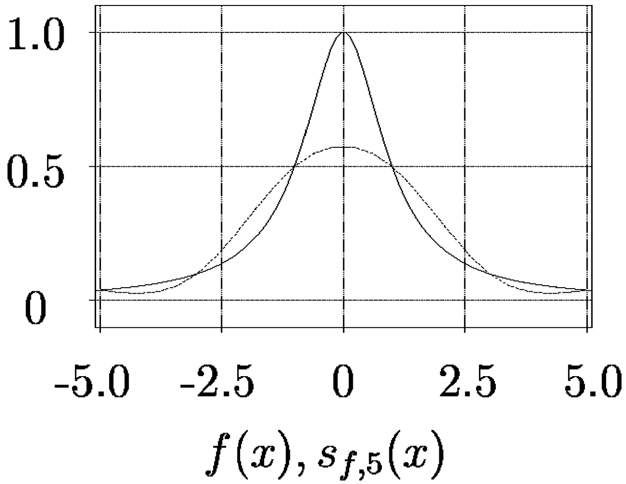
Aufwand: n Divisionen und $2n$ Additionen.

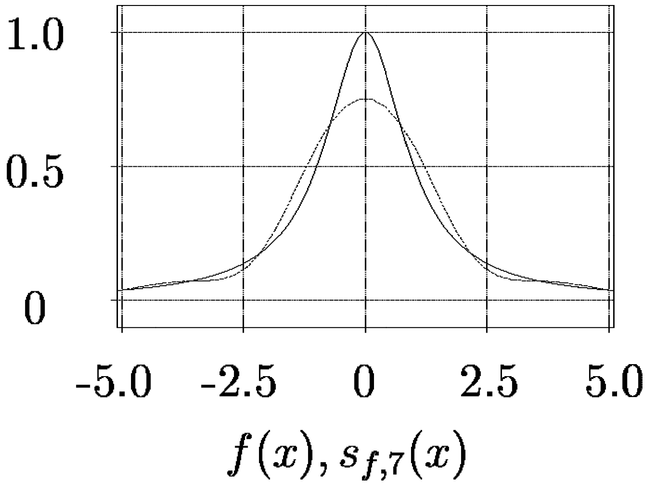


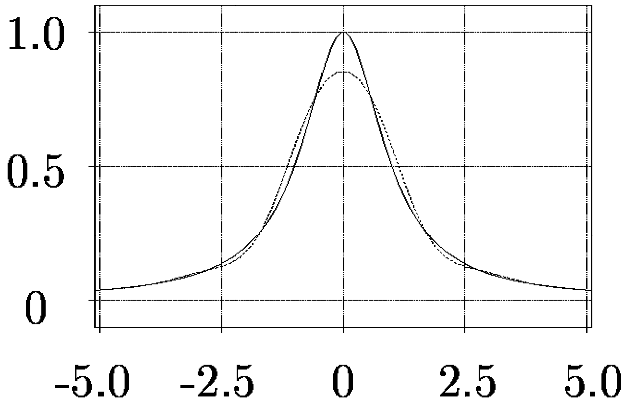
$P(x) - f(x)$



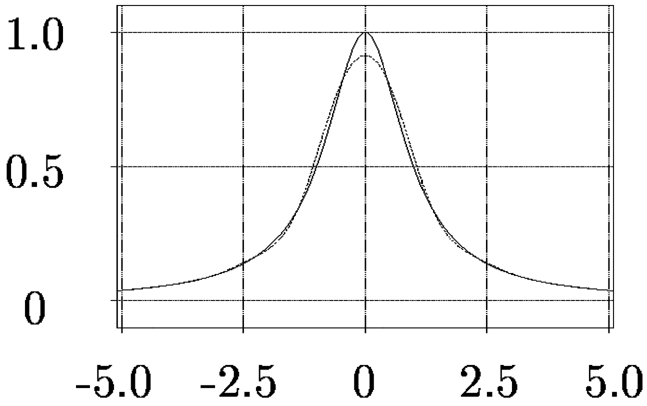
$\Phi(x) - f(x)$


1.0
0.5
0
-5.0
-2.5
0
2.5
5.0
$f(x), s_{f,5}(x)$

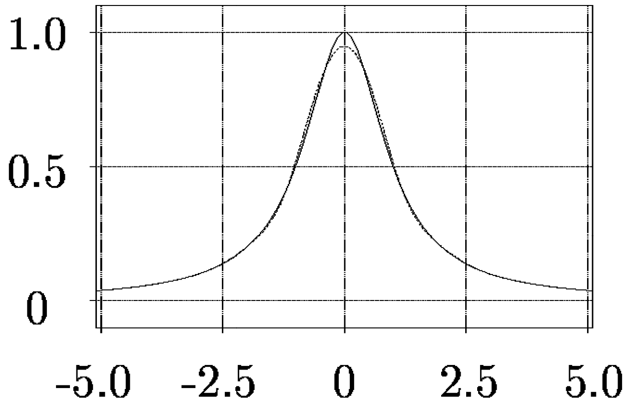

1.0
0.5
0
-5.0
-2.5
0
2.5
5.0
$f(x), s_{f,7}(x)$



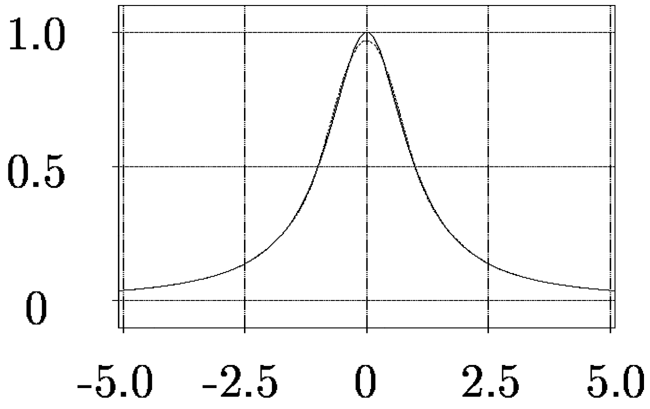
$f(x), s_{f,9}(x)$



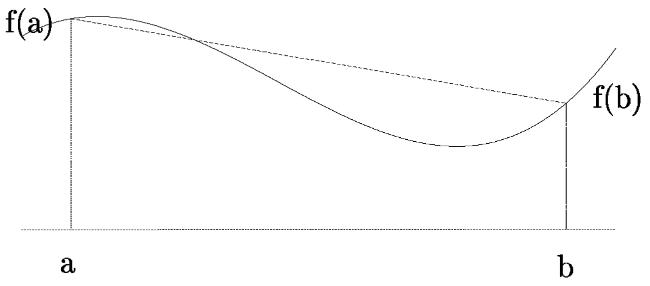
$f(x), s_{f,11}(x)$



$f(x), s_{f,13}(x)$



$f(x), s_{f,15}(x)$

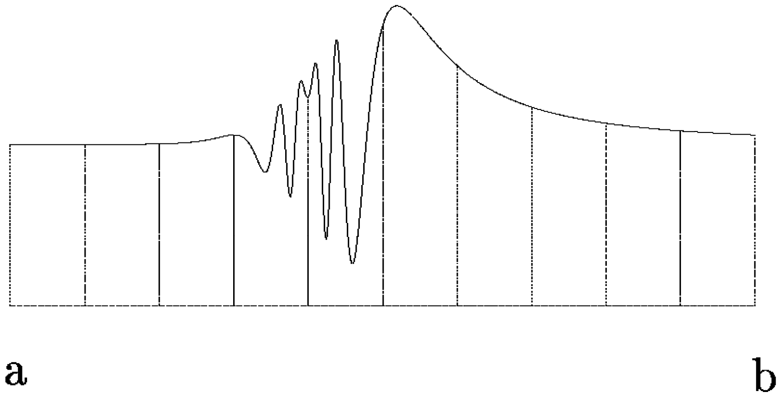

f(a)
f(b)
a
b

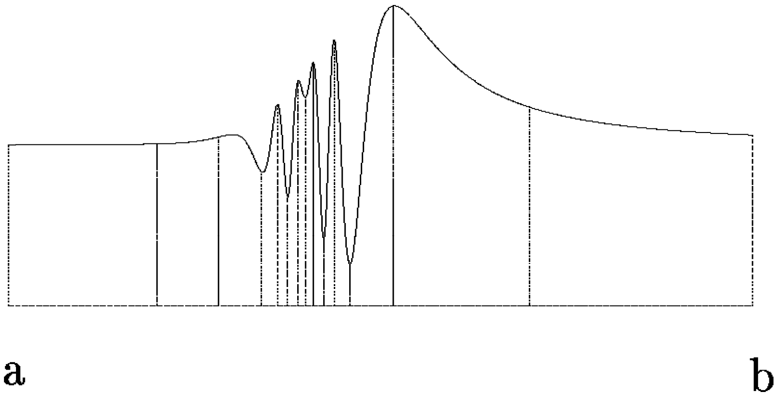
Geschlossene Newton-Cotes-Formeln

n	$s\sigma_i^{(n)}$							s	$R_n(f)$
1	1	1						2	$-\frac{h^3}{12}f''(\xi)$
2	1	4	1					6	$-\frac{h^5}{90}f^{(4)}(\xi)$
3	1	3	3	1				8	$-\frac{3h^5}{80}f^{(4)}(\xi)$
4	7	32	12	32	7			90	$-\frac{8h^7}{945}f^{(6)}(\xi)$
5	19	75	50	50	75	19		288	$-\frac{275h^7}{12096}f^{(6)}(\xi)$
6	41	216	27	272	27	216	41	840	$-\frac{9h^9}{1400}f^{(8)}(\xi)$
7	751	3577	1323	2989	2989	1323	...	17280	$-\frac{8183h^9}{518400}f^{(8)}(\xi)$
8	989	5888	−928	10496	−4540	10496	...	28350	$-\frac{2368h^9}{467775}f^{(10)}(\xi)$

Offene Newton-Cotes-Formeln

n	$s\bar{\sigma}_i^{(n)}$							s	$\bar{R}_n(f)$
2	1							1	$\frac{h^3}{3}f''(\xi)$
3	1	1						2	$\frac{h^3}{4}f''(\xi)$
4	2	-1	2					3	$\frac{14h^5}{45}f^{(4)}(\xi)$
5	11	1	1	11				24	$\frac{95h^5}{144}f^{(4)}(\xi)$
6	11	-14	26	-14	11			20	$\frac{41h^7}{140}f^{(6)}(\xi)$
7	611	-453	562	562	-453	611		1440	$\frac{5267h^7}{8640}f^{(6)}(\xi)$
8	460	-954	2196	-2459	2196	-954	460	945	$\frac{3956h^9}{14175}f^{(8)}(\xi)$


a
b


a
b

Orthogonalpolynome

$[a, b]$	$\omega(x)$	Name der Orthogonalpolynome
$[-1, 1]$	1	$P_n(x)$, LEGENDRE-Polynome
$[-1, 1]$	$1/\sqrt{1-x^2}$	$T_n(x)$, TSCHEBYSCHEFF-Polynome 1.Art
$[0, \infty)$	e^{-x}	$L_n(x)$, LAGUERRE-Polynome
$(-\infty, \infty)$	e^{-x^2}	$H_n(x)$, HERMITE-Polynome

ROMBERG-Integration

Wähle Schrittweitenfolge $\{ h_k \}$ *und Extrapolationstiefe* m.

```
for $k = 0$ to $m$ do
    {Berechne die Trapezsumme $T_{k0}$}
    $T_{k0} = h_k \left[ \frac{1}{2} f(a) + f(a+h_k) + \cdots + f(b-h_k) + \frac{1}{2} f(b) \right]$
    for $i = 1$ to $k$ do
        $T_{ki} = T_{k,i-1} + \frac{T_{k,i-1} - T_{k-1,i-1}}{\left( \frac{h_{k-i}}{h_k} \right)^2 - 1}.$
    endfor
endfor
```

Symmetrische Differentiationsformeln für die 1.Ableitung

$2k$	$s\beta_i^{(k)}$							s	$r_{2k}(f;\overline{x})$
2	-1	0	1					2	$-\frac{h^2}{6}f'''(\xi)$
4	1	-8	0	8	-1			12	$\frac{h^4}{30}f^{(5)}(\xi)$
6	-1	9	-45	0	45	-9	1	60	$-\frac{h^6}{140}f^{(7)}(\xi)$

Symmetrische Differentiationsformeln für die 2.Ableitung

$2k$	$s\gamma_i^{(k)}$					s	$\overline{r}_{2k}(f;\overline{x})$
2	1	-2	1			1	$-\frac{h^2}{12}f^{(4)}(\xi)$
4	-1	16	-30	16	1	12	$\frac{h^4}{90}f^{(6)}(\xi)$

Extrapolationsverfahren zur numerischen Differentiation

Wähle Schrittweitenfolge $\{h_k\}$ *und Extrapolationstiefe* m.

```
for $k = 0$ to $m$ do
    $D_{k0} = \dfrac{f(x+h_k) - f(x-h_k)}{2h_k}$
    for $i = 1$ to $k$ do
        $D_{ki} = D_{k,i-1} + \dfrac{D_{k,i-1} - D_{k-1,i-1}}{\left(\dfrac{h_{k-i}}{h_k}\right)^2 - 1}$
    endfor
endfor
```

Bisektionsverfahren

Es sei $f \in C[a,b]$, $f(a) < 0$ *und* $f(b) > 0$.

S0: *Setze* $a_0 = a$, $b_0 = b$ *und* $k = 0$.

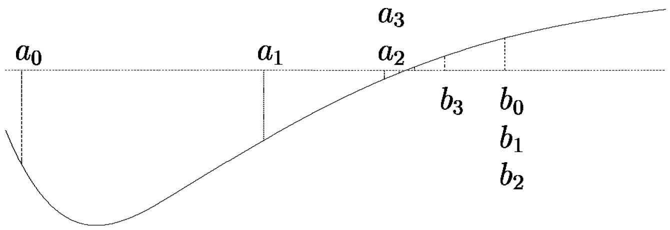
S1: *Berechne*

$$\xi = \frac{a_k + b_k}{2}, \quad \eta = f(\xi).$$

S2: *Für*

$$\eta \begin{cases} > \quad 0 \quad \mathit{setze} \quad a_{k+1} = a_k, \quad b_{k+1} = \xi, \\ = \quad 0 \quad \mathit{setze} \quad x^* = \xi, \quad STOPP, \\ < \quad 0 \quad \mathit{setze} \quad a_{k+1} = \xi, \quad b_{k+1} = b_k. \end{cases}$$

S3: *Setze* $k = k + 1$ *und gehe zu Schritt* **S1**.


a_3
a_0
a_1
a_2
b_3
b_0
b_1
b_2

NEWTON-RAPHSON-Verfahren 1. Art

Es sei $f \in C^1[a,b]$.

S0: *Wähle ein* x_0 *und setze* $k = 0$.

S1: *Berechne*

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}.$$

S2: *Setze* $k = k + 1$ *und gehe zu Schritt* **S1**.

NEWTON-RAPHSON-Verfahren 2. Art

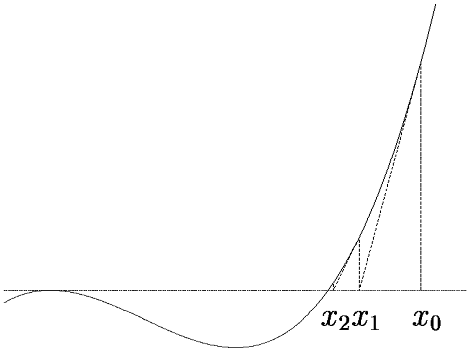
Es sei $f \in C^2[a,b]$.

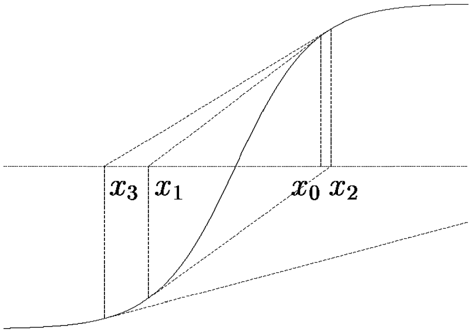
S0: *Wähle ein* x_0 *und setze* $k = 0$.

S1: *Berechne*

$$x_{k+1} = x_k - \frac{f'(x_k) \pm \sqrt{\left(f'(x_k)\right)^2 - 2f(x_k)f''(x_k)}}{f''(x_k)}.$$

S2: *Setze* $k = k + 1$ *und gehe zu Schritt* **S1**.


x_2
x_1
x_0


x_3
x_1
x_0
x_2

Regula falsi 1

Es sei $f \in C[a,b]$, $f(a) < 0$ *und* $f(b) > 0$.

S0: *Setze* $a_0 = a$, $b_0 = b$ *und* $k = 0$.

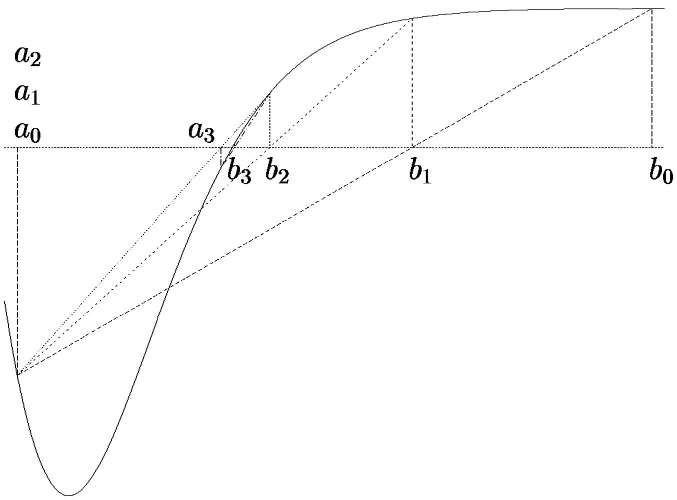
S1: *Berechne*

$$\begin{aligned} \xi &= \frac{a_k f(b_k) - b_k f(a_k)}{f(b_k) - f(a_k)} \\ &= a_k - \frac{b_k - a_k}{f(b_k) - f(a_k)} f(a_k), \\ \eta &= f(\xi). \end{aligned}$$

S2: *Für*

$$\eta \begin{cases} > & 0 \quad a_{k+1} = a_k, \quad b_{k+1} = \xi, \\ = & 0 \quad x^* = \xi, \quad STOPP, \\ < & 0 \quad a_{k+1} = \xi, \quad b_{k+1} = b_k. \end{cases}$$

S3: *Setze* $k = k + 1$ *und gehe zu Schritt* **S1**.


a_2
a_1
a_0
a_3
b_3
b_2
b_1
b_0

Regula falsi 2

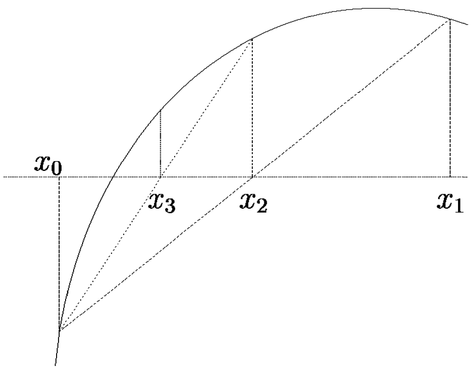
Es sei $f \in C[a,b]$, $f(a) < 0$ *und* $f(b) > 0$.

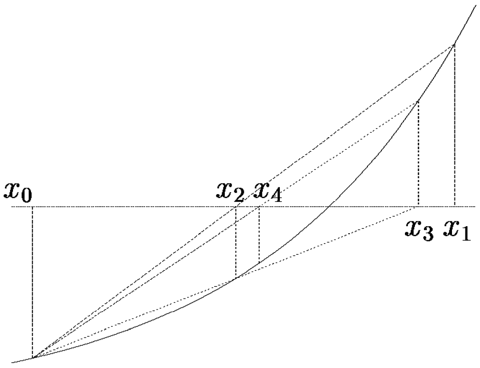
S0: *Setze* $x_0 = a$, $x_1 = b$ *und* $k = 1$.

S1: *Berechne*

$$x_{k+1} = x_k - \frac{x_0 - x_k}{f(x_0) - f(x_k)} f(x_k).$$

S2 *Setze* $k = k + 1$ *und gehe zu Schritt* **S1**.


x_0
x_3
x_2
x_1


x_0
x_2
x_4
x_3
x_1

Regula falsi 3 oder Sekantenverfahren

Es sei $f \in C[a,b]$.

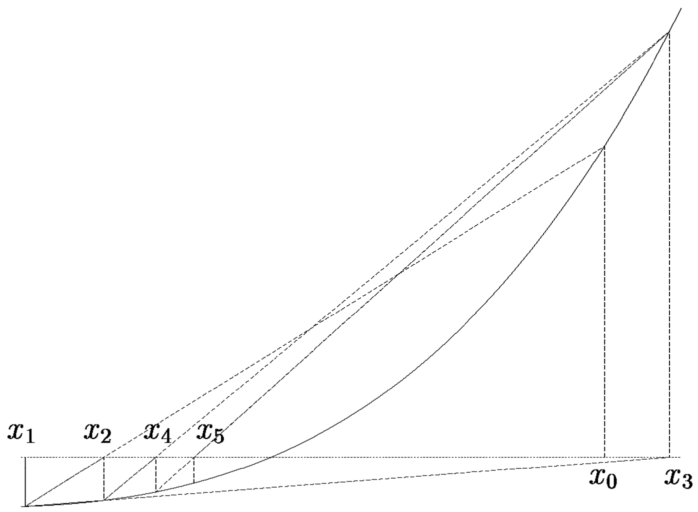
S0: *Wähle* $x_0 \in [a,b]$ *und* $x_1 \in [a,b]$ *und setze* $k = 1$.

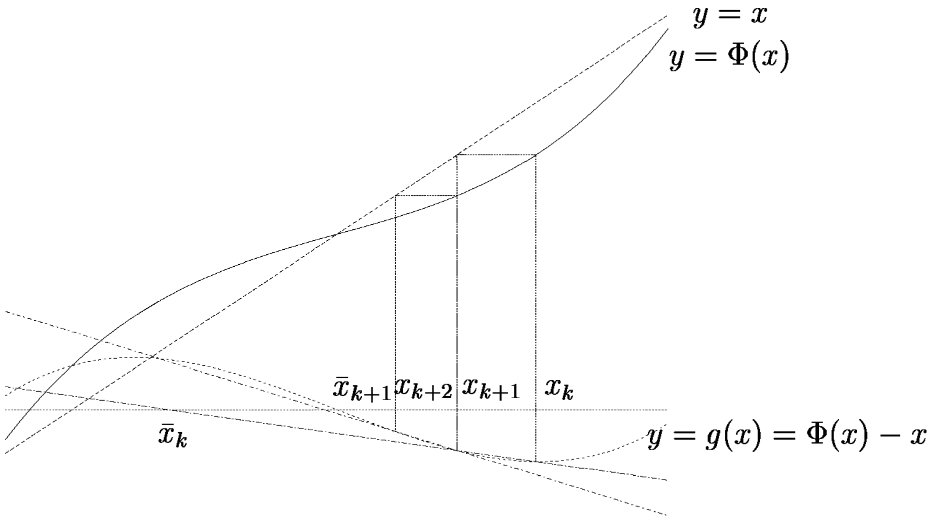
S1: *Berechne*

$$x_{k+1} = x_k - \frac{x_{k-1} - x_k}{f(x_{k-1}) - f(x_k)} f(x_k).$$

S2: *Setze* $k = k + 1$ *und gehe zu Schritt* **S1**.

5:11/6




$y = x$
$y = \Phi(x)$
$\bar{x}_{k+1}$
x_{k+2}
x_{k+1}
x_k
$\bar{x}_k$
$y = g(x) = \Phi(x) - x$

Hybridverfahren

Für eine Funktion f sei durch Φ ein Iterationsverfahren zur Nullstellenbestimmung mit einer Konvergenzordnung $p > 1$ gegeben.
Weiterhin sei $[a,b]$ ein Intervall mit $f(a)f(b) < 0$.
Die Funktion f erfülle alle Voraussetzungen des durch Φ gegebenen Iterationsverfahrens.
O.B.d.A. sei $f(a) < 0$ und $f(b) > 0$.

S0: *Setze*

$$a_0 = a, \quad b_0 = b, \quad x_0 = a, \quad k = 0.$$

S1: *Berechne* $\mu = \Phi(x_k)$.
S2: *Wenn* $\mu \notin [a_k, b_k]$, *so gehe zu* **S5**.
S3: *Berechne* $\eta = f(\mu)$. *Wenn*

$$\eta \begin{cases} > & 0, \quad d = \mu - a_k, \\ = & 0, \quad x^* = \mu, \ STOPP, \\ < & 0, \quad d = b_k - \mu. \end{cases}$$

S4: *Wenn* $d \leqq (b_k - a_k)/2$, *so gehe zu* **S6**.

S5: *Setze* $\mu = (a_k + b_k)/2$ *und berechne* $\eta = f(\mu)$.

S6: *Wenn*

$$\eta \begin{cases} > & 0, \quad a_{k+1} = a_k, \qquad\qquad\quad b_{k+1} = x_{k+1} = \mu, \\ = & 0, \quad x^* = \mu, \qquad\qquad\qquad \mathit{STOPP}, \\ < & 0, \quad a_{k+1} = x_{k+1} = \mu, \quad b_{k+1} = b_k. \end{cases}$$

S7: *Setze* $k = k + 1$ *und gehe zu* **S1**.